MATHEMATICS IN FRENCH EDUCATION

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France is reputed to have a highly centralised and complex educational system. Our goal is to make this system accessible to foreign readers. Therefore we will avoid precise details in our attempt to provide as faithful an overview as possible.

The text is divided into 6 sections:

- A general overview of the educational system
- Primary school
- *Collège* : lower secondary school
- *Lycée*: upper secondary school
- Tertiary level
- Teachers' training

1. GENERAL OVERVIEW OF THE EDUCATIONAL SYSTEM

The third Republic (1871-1946) and Jules Ferry, as Minister for Public Instruction, laid the true foundations of French education: non-denominational public education, free education and compulsory schooling from the age of six until the age of sixteen (since 1959).

The definition and implementation of educational policy is the responsibility of the government, except for the "fundamental principles of education" determined by the law voted for by Parliament.

The French educational system, which has by historical tradition always been highly centralised, is based on three principles:

- compulsory schooling up to the age of 16
- free public service
- non-denominational teaching (according to the 1905 law separating the Church and the State, schools are to be neutral and non-confessional).

The basic principles governing general education policy are set out in the 1989 framework law, which considers that education is a national priority. It fixes a number of principles governing school and university life in all its aspects, the functions and training of staff, the operation of schools and universities and assessment of the educational system.

Every child is entitled to attend a state school from the age of 3 up to the age of 16, but it is only compulsory from the age of 6. Indeed, most children attend nursery school (*école maternelle*), quite often from the age of 2 (in which case, they attend to nursery school for four years).

Elementary school (*école élémentaire*) lasts 5 years. The name of the 5 different levels is a survival of an old separation: CP (Preparatory Course) CE1, CE2 (Elementary Course) CM1 and CM2 (Middle Course). Primary school is now divided into two cycles: basic learning (CP-CE1) and consolidation (CE2-CM1&2). Nursery and elementary school form together what is known as primary school (*école primaire*).

Secondary education lasts 7 years, from the sixth to the terminal class (downwards). There are two levels: *Collège* - lower secondary education (4 years) and *lycée* - higher secondary education (3 or 4 years). At the end of the *lycée*, students sit a national examination, the *Baccalauréat* (Bac for short), that validates their secondary studies but also entitle them to enter any university.

The 1989 framework law on education made education the first national priority. It stipulates that four pupils out of five should reach *Baccalauréat* level, with other pupils reaching a minimum level of qualification corresponding to a CAP (Certificate of professional aptitude, a national diploma awarded to pupils having attended two years of training in a vocational *lycée*) or BEP (a vocational and national diploma qualifying pupils for a range of trades, awarded to pupils having attended two years of training in a vocational *lycée*. It is

common for this diploma to offer entry into courses leading to a vocational or technological *baccalauréat*).

Technological and vocational education has been renewed and developed since 1985. Renewal has included the modernisation of training and the development of links between schools and industry by creating training periods in industry and creating a vocational *baccalauréat professionnel*.

The 1993 law lays down the principle under which "all young persons should be offered vocational training before leaving the educational system, whatever level they reach".

Primary School (*école primaire*)	Nursery school (*école maternelle*) – 3 Years	Age 3-6
	Elementary school (*école élémentaire*) - 5 Years	Age 6-11
Lower secondary education (*collège*) - 4 Years		Age 11-15
Upper secondary education	*Lycée d'Enseignement Général et Technologique* - 3 Years	Age 15-18
	Lycée Professionnel- CAP /BEP – 2 Years	Age 15-17
	Lycée Professionnel BEP + *Baccalauréat Professionnel* – 4 Years	Age 15-19

Table 1. *General organisation of primary and secondary education in France*

Most schools are run by the state or a local authority (private schools usually have a contract with the state and a similar organisational structure) and most teachers are civil servants with tenured positions. In the public system, one cannot choose which school to attend, one is assigned depending on where one lives. This can be a reason for choosing a private school. The programmes including curriculum, content, objectives, activities, time organisation, etc., are national, although recently, some flexibility appeared concerning time organisation. However, essentially, France remains centralised and attached to the notion of equality of service for every citizen.

Normally, every child attends school until the end of the *collège*, (specific sections within *collèges* -SES or SEGPA- are designed for pupils with great difficulties).

By deciding to shift some of the powers and responsibilities hitherto held by the States over to territorial authorities in 1982, France has undertaken considerable decentralisation. This has radically changed the respective powers of the State departments and the territorial authorities.

However, the State continues to play a central role. It is responsible for ensuring public services are properly run, and for maintaining the coherence of education. Thus it continues to define the educational choices and curricula, and, as before, is responsible for recruiting, training and managing staff. The State also decides the status and operating rules for teaching establishments, and allocates the necessary teaching and administrative posts.

The State is organised as a strict hierarchy with different levels, summarised here:

- At the top level is the "Minister for National Education, and Research". The central administration comprises the general inspectorates, the private office, ten large Directorates

and a delegation, all responsible for implementing Government policy under the guidance of the Minister. Each directorate is responsible for planning and regulating its area of education;

Other minister are also involved in Education. The Minister for Agriculture and Fisheries is in charge of agricultural education, the Minister of Labour and Solidarity has an important role in vocational training, and the Minister for Sport and Minister of Culture contribute to the organisation of educational programmes for young people.

- At the regional level the administration is called *académie* or *rectorat*. It is an administrative sub-section of the Ministry of Education, and it's headed by a *recteur* who is appointed by decree. France is divided into 25 *académies* (plus 5 including Corsica and overseas territories) that roughly correspond to regional divisions. The *recteur* has powers which cover every level of education, whether primary, secondary or higher:
  - He must supervise the execution of the Minister's decisions and the implementation of all the legal provisions concerning education. He must also inform the Minister of the situation in his *académie*;
  - In agreement with local authorities, he has power over the organisation of primary education. He allocates posts among the various territorial authorities. Regarding organisation of secondary education, he participates in studies for prospective plans for training and investment, approves building programmes for schools and safety measures, and allocates certain educational materials.

- At the local level the administration is called *Inspection académique* and it corresponds to the department level. It's headed by an *inspecteur d'académie* who is appointed by decree. He has responsibilities at all levels of education excluding higher education. He or she is subordinate to the *recteur*, and is responsible for implementing and directing the ministry's educational policy in the department. He is assisted by a group of education inspectors. The *inspecteur d'académie* has both administrative and educational duties. His administrative role gives him the power to decide the opening and closing of classes, and where to place schoolteachers. He also has the power to manage primary school staff and to inspect schools. Finally, he plays an important part in the preparation of the school year and in the definition of the department school network. His educational remit consists of introducing innovation in all areas and informing teachers and head teachers in primary education. He has a more specific role in lower secondary education (colleges), in planning and directing schools (operating supervision) and in organising school life (participation in the projected educational and investment plans, organisation of examinations, and school attendance). His educational role is the same as in primary education.

The 1982 and 1983 laws regarding the decentralisation of government considerably expanded the role of local authorities. The territorial authorities (which are elected) and the State share the general responsibility for management and administration of schools. Each local authority is responsible for the schools at every education level:

- *Communes* (city councils) are in charge of primary schools. They are responsible for the building, maintenance and administrative control of primary schools, with decisional powers over their creation and budget management. However, the State representative must agree to create a school or a class.

- *Départements* (France is divided into 96 *départements*) are in charge of lower secondary education. They are responsible for *collèges* (state or private secondary schools of lower level for all pupils having finished their primary education). Schooling in these institutions lasts four years, with sixth, fifth, fourth and third classes in ascending order. These territorial authorities deal with school transport and the maintenance of building of *collèges*.

- *Régions* (France is divided into 22 *régions*) are in charge of upper secondary education. They are responsible for *lycées* (state or private secondary school), and for specialised

establishments. There are two categories of *lycées*: those for general and technological education (LEGT), and those for vocational studies (LP). The region is responsible for the building and maintenance of the *lycées* subsidises the daily operational costs and has powers regarding the implementation of vocational training. Local communities are now involved in making decisions regarding school planning.

Generally, the territorial authorities are responsible for building, re-building, extensions and extensive repairs and equipment; and the State is responsible firstly for educational and staff costs and secondly for organisation, timetables and curricula in schools.

2. PRIMARY SCHOOL

Ecoles maternelles ("nursery school") offer schooling for all children from the age of three (and very often two) until compulsory school, that is to say the age of six. Admission to these institutions is on request by the family.

The framework law of 1989 laid down the main guidelines for a new school policy. New methods of organising and operating nursery and primary schools were applied in 1990. In 1991, a Ministry document outlined the key aspects of the new policy for nursery and primary schools, and detailed the skills pupils must acquire during each cycle.

The general objective of nursery school is to develop the child's full potential in order to shape his personality and give him the best chances of success in primary school and in life, preparing him for later learning, and for life in society.

Primary school is divided into three cycles:

- **Cycle 1: Initial learning**. This corresponds to the three years of nursery school. However, the last year of nursery school (*grande section*) is also the first year of cycle 2 and therefore belongs to both cycles.
- **Cycle 2: Basic learning**. This corresponds to the last years of nursery school and the first two years of primary school (CP preparatory course, CE1 elementary course1)
- **Cycle 3: Consolidation**. This corresponds to the last three year of primary school (CE2 elementary course1, CM1 and CM2 middle course 1 and 2)

Pupils are allowed to progress at their own speed, according to their abilities and maturity. For this reason, teaching methods may vary, the aim being to implement the teaching strategy most suited to each pupil. The weekly duration of classes is 26 hours. The curriculum is compulsory but the choice of teaching methods is up to the teacher.

The teachers of the various cycles analyse together the progression of each pupil: the analysis is based on periodic assessments carried out by the teachers during each cycle, testing what the pupil has learned. A systematic assessment of all pupils is organised at the beginning of the consolidation cycle (8 years of age) and at the beginning of the *collège*. Otherwise there is no examination at the end of primary school.

2.1 Ecole maternelle : initial learning

Nursery school focuses on initial learning, with the exploration of the child's environment, manipulation and play being the main activities. Five main objectives have been set out:

1. Language at the heart of learning
2. Live together
3. Act and express oneself with the body
4. Discover the world
5. Sensitivity, imagination and creation

There is no 'real mathematics', but young children begin to use a mathematical way of thinking. They build their first knowledge about numbers, geometrical shapes and magnitudes and get acquainted with structuring space and time. The activities they are

involved with also contribute to developing logical thinking in terms of comparing, classifying, organising and using symbols.

Mastering ordinary space means knowing how to locate oneself and other people or objects in it, organise movements, and knowing how to describe them from different points of view (oneself or others'). This progressive mastering evolves along with the use of an adequate vocabulary.

Mastering time means mastering two ideas: chronology and duration.

Work on shapes is organised through games (jigsaws, paving, assembling, portrait game, etc.) which encourage children to identify common shapes (square, circle, triangle, etc.) as well as some of their characteristics (expressed in terms such as: *straight, curved, pointed, etc.*

Basic magnitudes (mainly length, mass and capacity) are essentially seen though comparing, classifying and organising activities.

First learning about numbers at *école maternelle* take into account the results of numerous research works in psychology and didactics. Children are taught the verbal numerical chain (at least up to 30) and use it to count things. Number becomes a tool for checking quantities, and remembering them, in order to anticipate the result of certain actions on these quantities (augmentation, diminution, sharing, etc.), even if counting is not a goal in itself at this level. Numbers are mainly expressed orally, but children start recognising and writing figures.

2.2 Cycle 2: Basic learning

- Generally speaking, the purpose of **cycle 2**, concerning basic learning, is to ensure that basic knowledge has been acquired and structured: oral and written expression, reading and mathematics. Weekly time slots are: French: 9 to 10 hours; **Mathematics: 5 to 5,5 hours**; Living together: 0.5 hours; Discovery of the world: 3 to 3.5 hours; Foreign or regional language: 1 to 2 hours; Artistic education: 3 hours. Physical education and sport: 3 hours. Within this timetable, 2.5 hours must be dedicated every day to reading and writing.

There are two priorities:

- problem solving is the source, the goal and the main support of mathematical learning
- mental arithmetic, for which the goals are memorisation of elementary results and capacity to develop non-memorised results (elaborate computations)

A favoured teaching strategy is initial research into concrete situations, leading to verbal exchange, but the importance given in the classroom to written tasks remains an obstacle.

The syllabus is organised around five domains:

1. Exploiting numerical data

Two categories of problems have to be distinguished depending on whether an *expert resolution* is expected or a more *personal* one, in which pupils can take initiatives when no direct application of the notion at stake is visible.

2. Integers

The main goal is that pupils learn to control the principles of decimal numeration and be able to correlate the value of a digit according to its position in the decimal writing of the number, from counting experiences using grouping or exchanges (by tens or hundreds), for numbers under 1,000. They must also be able, for such numbers, to associate decimal writing and oral designation, to compare them, to master the organisation of the written digits sequence and to do basic arithmetic (essentially finding doubles and halves).

3. Arithmetic

Mental computations are favoured, in order to learn tables of sums and to prepare an organised treatment of simple subtractions and multiplications. Written computing algorithms are only seen for sums. Calculators can be used when solving problems.

4. Space and geometry

Following cycle1, the mastering of ordinary space is the main goal. A first approach of 2D and 3D figures, beyond visual perception, is made possible through the study of some properties (alignment, right angle, symmetry axis, equality of lengths) and the use of some tools (ruler, templates for right angles).

5. Magnitudes and measurement

Length, mass and capacity are the only magnitudes seen at this level. They are put into context by comparing objects (directly or indirectly), then measuring them with use of a standard unit. Common units of measure are introduced: cm and m; kg and g, l. New notions concerning the location and measurement of time are introduced: use of a calendar, chronometer, expressing length of time in days, hours, minutes, seconds.

2.3 Cycle3: Consolidation

- The consolidation cycle 3 is characterised by two features: reinforcement and consolidation of knowledge acquired in the previous cycle, and the gaining of broader knowledge and development of greater rigour in working methods. Weekly time slots are:

- French language and literature and Human education: 12 hours (literature (talking, reading, writing): 4.5 to 5,5 hours; reflective observation of French language (grammar, conjugation, spelling, vocabulary): 1.5 to 2 hours; foreign or regional language: 1.5 to 2 hours, History and geography: 3 to 3.5 hours; Collective life: 0.5 hours)
- Scientific education: 8 hours (**Mathematics: 5 to 5.5 hours**; Experimental sciences and technology: 2.5 to 3 hours)
- Artistic education: 3 hours
- Physical education and sport: 3 hours

As in cycle 2, there are two priorities:

- problem solving is the source, the goal and the main support of mathematical learning
- mental arithmetic for which the goals are being able to memorise elementary results and develop non-memorised results (elaborate computations)

New general abilities are aimed at, in particular concerning:

- organising problem-solving in steps, and the reasoning abilities necessary for it
- formulating in a more rigorous way, arguing about the validity of a solution and pointing out errors in a solution.

The syllabus is organised around six domains:

1. Exploiting numerical data

At the end of cycle 3, pupils must be able to solve most problems involving the four operations on integers, addition and subtraction of decimal numbers and multiplication of a decimal number by an integer.

Using appropriate reasoning (not general techniques), pupils are asked to solve problems on proportionality. This involves properties such as linearity or the multiplicative coefficient, which remain implicit.

There is also a first initiation into the reading and building of tables, diagrams or graphs.

2. Integers

The main goal is, as for cycle 2, that pupils control the principles of decimal numeration and are able to correlate the value of a digit according to its position in the decimal writing of the number whatever its size. They must also be able, for certain numbers, to

associate decimal writing and oral designation, to compare them, to master the organisation of the written digits sequence, to situate them on a graduated line, and to do basic arithmetic (find doubles, halves, quadruples, quarters, …, and recognise multiples of 2 and 5).

3. Fractions and decimal numbers

Fractions are limited to what is necessary to know in order to understand decimal numbers: no competence concerning operations and comparison is at stake. Fractions are defined with reference to the division of a unity: 4/3 is seen as "four times the third of the unity". Concerning decimal numbers (like integers), the main goal is that pupils master the principles of the representation with the comma and are able to determine the value of a digit according to its position. The emphasis is on the question of ordering (comparing, finding an interval, inter-classifying), pointing out that some properties are breaking with what is known about integers. Positioning numbers on a graduated line allows a better appropriation of these properties.

4. Arithmetic

Mental computing is still favoured, in order to learn tables of sums and multiplications and to prepare an organised treatment of additions, subtractions, multiplications, and some divisions (easy cases of integers and decimal numbers). Pupils are expected to master the written algorithms for the four operations on integers as well as the addition, subtraction and multiplication of decimal numbers by an integer. Pupils are encouraged to use a calculator, in particular for problem-solving, moreover, some functionality of ordinary calculators must be acquired (constant factor, memory).

5. Space and geometry

Mastering ordinary space is completed by a first study of its representation (maps, plans). The study of geometrical properties (alignment, perpendicularity, parallelism, equality of lengths or angles, axis of symmetry) and use of tools (ruler, set square, compass) allow pupils to characterise some solids (cube, right-angled parallelepiped) and some plane figures (triangles, square, rectangle, rhombus and circle). Work on enlargement or reduction of figures leads pupils to see which geometrical properties are preserved and connects with proportionality (for segments' length).

6. Magnitudes and measurement

Using magnitudes encountered in cycle 2, pupils learn to make rough estimations (order of magnitude), to apply legal units of measure and understand their relationships, and to make calculations using these magnitudes (perimeters and duration essentially).

Two new magnitudes are studied:

- the notion of area, which is clearly distinguished from the notion of perimeter; some common units of measure are introduced and their relationship learnt (cm^2, dm^2, m^2, km^2); work on the area of a rectangle.
- the notion of angle is made more precise: comparison, reproduction, measurement with the use of fractions of the right angle.

3. *COLLÈGE* : LOWER SECONDARY SCHOOL

In 1963, the increased social demand for education and the population growth led to the creation of a new type of school for children of 11-12: the *collège*. The "Haby law" (1975), which completed the "democratisation", set up the *collège unique* for all children arriving from primary school.

All children having completed their primary education have the right to be admitted into the *collège* in their twelfth year at the very latest. Those of them who have not acquired the skills taught in primary school may be admitted to a specialised section (SES or SEGPA).

- The sixth class corresponds to the ***observation and adaptation cycle*** of secondary education
- The fifth and fourth classes correspond to the ***consolidation cycle***
- The third class corresponds to the ***orientation cycle***

Each school has a minimum allowance of 26 hours weekly. The definition of maximum-minimum timetables per subject allows a diversified selection of courses based on pupils' particular interests and needs. The subjects are: French, Mathematics, First and second modern foreign languages, History - Geography - Civic life, Earth Sciences, Physics-Chemistry, Technology, Artistic education (Plastics arts and music), Physical education and Sports. Pupils can also choose among optional subjects.

Pupils have, in theory, access to computer facilities in their schools, in order to familiarise themselves with them. There is an optional degree in computer science offered, in the last class of the *collège*.

The *collèges* must give pupils a general education that will enable them to acquire basic knowledge, and prepare them for the training opportunities at the end of the third class.

Furthermore, technological education is available to pupils in fourth and third class.

At the end of the third class, pupils are awarded a national diploma, the *Brevet des Collèges*, evaluated through a final examination and marks given during the last two years of the *collège*.

After the third class, pupils can continue in a *lycée* for general and technological education (*lycée d'enseignement général et technologique*), choose full-time vocational education in a vocational *lycée* (*lycée professionnel*) or enter apprenticeship in an apprenticeship centre.

The syllabus for *collège* is presently under revision, in order to improve the synchronisation of this with both the *école élémentaire* and *lycée*. However, tit is likely that only minor revisions will be made. The syllabus presented here was published in 1996.

It declares that mathematics is a "discipline of general formation" where "problem solving, modelling of situations and progressive learning of proof" allow pupils to "become conscious step by step of what a real mathematical activity is". At *collège* level, mathematics must appear both as a discipline providing tools useful in everyday life and other domains, and as a discipline with its own autonomy. However, the question of the relationship between mathematics and other disciplines remains open: the pedagogical device named *Itinéraires de découverte* (imposing trans-disciplinary activities) demonstrated the difficulties that arise when teachers of different disciplines collaborate, even if some interesting experiments have taken place. The progressive nature of mathematical learning, time pressure and the amount of contents to be taught may partly explain these difficulties.

During the 4 years of *collège*, organised in 3 cycles, the syllabus is divided into 3 headings:
- Geometrical work
- Numerical work
- Data organisation and management. Functions.

The syllabus under discussion (to be applied in 2005) plans to set up a new division, inspired by one at *école élémentaire* level. The main visible change is a decreased emphasis on geometrical transformations, with possibly the focus shifted to more important modifications concerning geometry.

3.1 Observation and adaptation cycle (sixième)

The weekly timetable is 4 hours.

For the main part, the content is similar *to école élémentaire*.

1. Geometrical work

The study of "simple" figures begun in *école élémentaire* is deepened. Tools derived from axial symmetry allow to better structure what has been acquired in *école élémentaire* (e.g. the study of the properties of isosceles and equilateral triangles, rectangle, rhombus or square), or to acquire new knowledge (e.g. the characterisation of the perpendicular median of a segment or the bisector of an angle).

Activities of recognition, and the drawing or reproduction of "simple" plane figures are of great importance. Work on the right-angled parallelepiped begun in *école élémentaire*, is enriched by the making of patterns and representations in perspective, using orthographic projection. Magnitudes are now in the geometry section. The distinction between area and perimeter is discussed again. The notion of volume is also introduced, along with the units of measure. The length of a circle, the area of a right-angled triangle and the volume of a parallelepiped are taught.

2. Numerical work

Knowledge about integers and decimal numbers is consolidated, especially regarding arithmetic (mental, manual or with a calculator), with specific work on the order of magnitude. Multiplication and division are extended to decimal numbers.

Fractions, introduced during *école élémentaire*, are now seen as quotients of two integers and become numbers.

A first initiation into the solving of equations (without the use of a letter to denote the unknown) is given. Formulae with letters are used in familiar situations.

Negative numbers are introduced, especially in relation to representation on a graduated line.

3. Data organisation and management. Functions

Although not explicitly mentioned, proportionality and statistics are central elements of this part of the syllabus: topics include percentage rates, calculations of magnitudes and changes of units (of length or area), and statistical listing with the use of tables or graphs.

The main goal is to solve problem involving proportionality through reasoning; this involves using in a new context the multiplication of an integer by a fraction.

The new syllabus should help clarify this area of the study.

3.2 Central cycle (cinquième and quatrième)

The weekly timetable is 3.5 hours (but it can be up to 4.5 hours if time is taken from *the itinéraires de découverte*). Most teachers consider this an insufficient amount of time.

New contents are introduced and initiation to proof is strengthened in order to achieve "a complete elaboration of a deductive thinking in simple cases".

1. Geometrical work

This domain is particularly used to develop competence in proof.

By carrying out experimental activities (freehand drawing, or with instruments, or a computer) related to central symmetry (*cinquième*) pupils are able to prove some of the properties of parallelograms (and of rectangles, rhombus and squares) as well as the angular characterisation of parallelism. In *quatrième*, translation is defined using the parallelogram (vectors are not introduced until *troisième*).

The triangle is a figure that leads to new areas of learning: sum of the angles, triangle inequality, construction problems, circumcircle (in *cinquième*), theorem about the middle points of the sides, Thales theorem (implication only) and Pythagoras theorem, remarkable straight lines in a triangle, circumcircle in the case of a right-angled triangle, cosine (in *quatrième*).

The notions of tangent to a circle and of distance from a straight line to a point are also introduced.
The study of solids is mainly pursued on experimental bases, by exploiting results from plane geometry: right prismoid, cylinder of revolution (in *cinquième*), pyramid and cone of revolution (in *quatrième*). Moving from the object to its various representations constitutes the main body of work, which is also an occasion to build mental images linked to situations of parallelism and orthogonality in space.
Learning about magnitudes concerning areas and volumes of plane figures and solids from a geometrical viewpoint.

2. Numerical work

Problem solving is the main goal of this part of the syllabus.
Arithmetic (4 operations) on integers, decimal numbers or fractions is extended to include negative numbers. Power with an integral exponent are introduced. The use of letter is also extended, especially in *quatrième*, with a double perspective: numerical calculations with use of letters, development of a simple letter expression, modelling with equations of the first degree with one unknown.

3. Data organisation and management. Functions

Proportionality constitutes one of the two important issues in this section. Location on a graduated axis and in the plane, worked in *cinquième*, leads in *quatrième* to a graphical characterisation of proportionality. This is used in problems with scales, uniform movement or average speed, percentage, change of units of measure, in particular (in *quatrième*) with quotient magnitudes.
The second issue concerns the initiation to statistics: reading, interpretation and representation of statistical series, notion of class, size and frequency (*cinquième*) cumulative size and frequency, weighted mean (in *quatrième*), with an initiation to spreadsheet software.

3.3 Orientation cycle (troisième)

The weekly timetable is 4 hours.
Initiation to proof is pursued, pupils "having to build and write proofs."

1. Geometrical work

This domain remains essential in order to develop pupils' abilities in proving.
The triangle is still the figure from which new knowledge is issued: in right-angled triangles new trigonometric relationships are introduced (sine, tangent); the Pythagorean formula is given with the related the formula for the distance between two points with use of their co-ordinates; Thales theorem is extended to the reciprocal implication.
The study of vectors is limited to the notions of sum (linked to the composition of translations) and co-ordinates.
The study of transformation is completed by rotations, which lead to some new properties of regular polygons.
The study of solids is mainly pursued on an experimental basis. It focuses on plane sections of already known solids and on the sphere, including its area and volume.

2. Numerical work

Problem solving is still the main goal of this part of the syllabus.
Introduction of the square root, elementary calculations with square roots (products, quotients), the notion of irreducible fraction, introduction of the notions of divisor, (greatest) common divisor (nothing on prime numbers).

Calculations with letters are further developed. Pupils must be able, in simple cases, to factorise an expression and to use common identities. They model situations leading to equations and inequations of first degree with one or two unknowns.

3. Data organisation and management. functions

Proportionality introduced in *école élémentaire*, is now linked to the linear function, which is studied along with affine functions (in particular graphical representations). It is used in relation to product magnitudes. Pupils investigates the effect of a reduction or an enlargement with a factor *k* on areas and volume. Initiation to statistics is pursued with the notion of median and the idea of dispersion. The study of spreadsheet software is continued.

4. *LYCEE*: UPPER SECONDARY SCHOOL

Upper secondary education is divided into three distinct branches: general education, technological education, and vocational education. In theory, there are only two types of *lycée*: the *lycées d'enseignement général et technologique* and the *lycées professionnels*.

Roughly 68% of an age class take the *baccalauréat*, in a general (34%), technological (21%) or vocational (13%) stream.

4.1 Lycée d'enseignement général et technologique

General and technological education lasts for three years (in the order *seconde*, *première* and *terminale*) and leads to the general and technological *baccalauréat* which is a national school diploma leading to higher education.

- The *seconde* (first class of the *lycée*) constitutes the ***determination cycle***. It is a class for undifferentiated general and technological education. At the end of the second class, students select either the general or technological branch and specialise in some subjects. They may also repeat the class, or in some rare cases, leave for a vocational *lycée professionnel*.
- The *première* and *terminale* (last two classes of the *lycée*) constitute the ***terminal cycle***, which prepares students for the *baccalauréat*. Students are divided among different classes according to the series or subjects in which they have chosen to specialise.

In the general branch, there are three series: Literature (L), based mainly on French, philosophy and modern languages; Economics and Social Sciences (ES), based mainly on economics and social sciences; Sciences (S) based on mathematics, physics and natural sciences.

In the technological branch, there are four series: Tertiary Sciences and Technology (STT), Industrial Sciences and Technology (STI), Laboratory Sciences and Technology (STL), Medical and Social Sciences (SMS).

As well as specialising in one of these 7 series, students must choose between different options (for instance between biology/geology and engineering sciences in the *série* S). In *terminale* of the general *séries* ES, L or S, they also have to choose optional classes, which cover additional topics in one of their main subjects[1] (for instance *Série S* specialising in mathematics will correspond to the best-trained students in mathematics).

Table 2 gives a rough estimate of the repartition of students in the various general and technological series, based on data concerning students in the *terminale* class in 2000/2001 in public and private education, as well as the percentage of girls in each series:

[1] Except in *série* L, where it is possible to choose an option about Arts.

Série	S	L	ES	STI	STT	STL	SMS
Percentage	31,7%	13,3%	19,3%	9,5%	20,1%	1,6%	4,6%
Percentage of girls	43,2%	82,4%	63,1%	7,6%	63,3%	55,0%	95,7%

***Table** 2: repartition by séries and girls' distribution*

The weekly timetables in upper secondary education ranges from 29.5 to 31.5 hours. Courses in the second class comprise core subjects (French, Mathematics, Physics-Chemistry, Technology of automated systems, Modern language 1, History-Geography, Physical education and Sports), compulsory optional subjects and a choice of subjects at the students' discretion.

Curricula and subjects in the terminal cycle change according to the *séries*. Curricula in all the technological *séries* provide for a general education, comprising two or four hours per week (depending on the subjects) of mathematics, French, philosophy, history and geography, one modern language and physical education. This general education is accompanied by specialised courses corresponding to the series chosen. The number of hours per week depends on the option chosen in the series.

In tables 3 and 4, we show the amount of mathematics and the weight of mathematics in the *baccalauréat* for the 3 series in general education.

Class level	*Série*	Timetable	Mini-maximum
Seconde	all	C:3h - M:1h - AI *:1h	4h - 5h
Première	L	C:1h - TD:1h - O *:3h	2h - 5h
Première	ES	C:2.5h - TD:0,5h - O *:2h	3h - 5h
Première	S	C:4h - TD:1h	5h
Terminale	L	O *:3h	0h - 3h
Terminale	ES	C:4h - S *:2h	4h - 6h
Terminale	S	C:4,5h - TD:1h - S *:2h	5.5h – 7.5h

***Table** 3: Current mathematics timetables in general high school*[2]

Série	Written test duration	Coefficient
L	1,5h (taken in *première*) 3h (if option in *terminale*)	2 out of 38 4 out of 34
ES	3h	5 out of 37 (+2 if speciality)
S	4h	7 out of 38 (+2 if speciality)

***Table 4**: Weight of mathematics in the baccalauréat*[3]

[2] In this table, C means " Cours", M : "Modules", TD : " Travaux dirigés", O : "Option", S: " Specialité" and AI: "Aide individualisée". "Cours" denote whole class activities, "Modules" and « Travaux dirigés », half-class activities, "Option", optional courses, "Specialité", the compulsory speciality courses, "Aide individualisée", remedial activities organised for groups of at most 8 students. '*' means that it does not concern all students.

As can be seen, even in the orientation S, the weight of mathematics is only about 20%.

In France, the teaching of high school mathematics has to face important changes. First of all, there is increasing competition with other disciplines, which obliges mathematics teaching to justify the role it plays in the curriculum and its educational value. Becoming a mass teaching, it must also face the increased heterogeneity of students and adapt to their cultural diversity. It must take into account the evolution of mathematics, the relationships between mathematics and other disciplines, and between mathematics and society, and it must try to address the new needs that result from these. It must take into account technological development and its effect on mathematical practices as well as on the means and forms of learning and teaching. It must face hour reductions which, when cumulated, mean that today's students, on average, lose one year of schooling if compared with students who entered secondary education fifteen years ago. It must finally help to fight against the increasing student disaffection with regard to scientific careers. Under these conditions, the mathematical and didactic organisations that the high school system has progressively built become more and more problematic.

It would take too long to detail precise curricula in mathematics for every *série*. Here we present the succinct content of *seconde* and the main trends in the latest changes to the general *séries*[4].

The curriculum of the *seconde* class is divided into three main parts:

- Numbers and functions divided in nature, and representation of numbers, rough estimation, manual and machine calculations, prime numbers, order and absolute value, notion of function, qualitative study, reference functions, algebraic formula, equations and inequations (mostly first order) algebraic and graphic resolution, modelling.
- Geometry divided in space geometry, two-dimensional configurations, analytical reference system and vectors.
- Statistics divided in fluctuation of sampling, simulation, descriptive statistics.

First, we would like to stress that the new curriculum does not pretend to implement radical change, as was the case with the new mathematics reform in the early seventies, or with the counter-reform in the early eighties. But, as is stressed in the introductions of the different high school syllabuses, school is questioned by the continuing scientific, technological and cultural evolution and must regularly reconsider its objectives in the light of this evolution.

Amongst the recent changes in the mathematical curricula, we find the following especially significant: 1. the increased emphasis on statistics, 2. the increased differentiation of the curriculum, according to the different possible orientations and the introduction of new themes, 3. the increased importance given to computer and information and communication technologies, 4. the increased focus on the links between mathematics and other scientific disciplines.

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[3] The written tests, for orientations ES and S, are partially different for students having chosen the speciality course. Tests include two exercises and one problem, and one of the exercises is specific for speciality students.

[4] For more information see Artigue (2003) The teaching of mathematics at high school level in France: designing and implementing the necessary evolution, In A. Gagatsis & S. Papastavridis (eds.), *Proceedings of the third international mediterranean conference on mathematics education, Athènes, janvier 2003*, Athènes: Hellas, pp. 9-34; on which this presentation is based.

4.1.1 An increased emphasis on statistics

This is the one of the major changes. The previous secondary curriculum in *collège* and *lycée* covered statistics, but only in a limited way (a basic introduction to descriptive statistics). Moreover, *collège* and *seconde* teachers tended to skip this chapter if they felt that they might not have time to cover the whole syllabus. From this point of view, the new syllabus for the *seconde* class published in 1999 constitutes a small revolution. In the mathematics programme, the part devoted to statistics, which according to the accompanying documents corresponds approximately to 1/8 of the total mass of the course, does not reduce simply to descriptive statistics. It includes a component of inferential statistics through the notion of fluctuation of sampling and simulation.

The language of probability introduced in *première* S, ES and in the option of *première* L, formalise the naive language of chances used in *seconde*; probabilistic computations enable students to explain some observed phenomena.

4.1.2 An increased differentiation of the curriculum according to the different possible orientations

The syllabuses for the non-scientific *séries* have been often defined by reduction. The evolution process started with the preceding *lycée* syllabuses, set up in the early nineties, especially for the *série* ES, and culminated with the syllabuses published in 2000, which introduced of new mathematical fields in the non-scientific *séries*.

The programme of *première* L focuses on mathematics used in a visible way in today's society: tables of numbers, percentages, some statistical parameters, graphical representations. This syllabus is organised in four parts: quantified information, statistics, examples of types of growth, opening activities. This last part, which is not taken into account in the *baccalauréat*, includes two topics: obtaining geometrical figures (such as the flake of Von Koch) per iteration, and analysing and producing plane pavements. This is a preparation for the syllabus of the option of *terminale* L, which, in geometry, includes the following topics: golden section and regular pentagon, perspective.

The option of *première* L has two main parts of geometry and analysis, to which some elements of combinatorics are added. If analysis is quite traditional, the geometry, organised around geometrical constructions with rule and compass, and introducing the concept of constructible number, proposes a more original approach, allowing students to tackle some famous historical problems such as the duplication of the cube, the trisection of the angle and the quadrature of the circle. The interest of such a historical perspective is pointed out in the presentation of the option. Unfortunately, it seems that this aspect will disappear in the new syllabus currently under discussion.

During the 1990s, the processing of quantified data became an important part of the *série* ES syllabuses. It remains so, and the decision has been made to institute a style of mathematics teaching consistent with and adapted specifically to the orientation ES. An option offered in *terminale* introduces some elements of matrix algebra and an innovative approach to graph theory. The syllabus description of this topic is as follows: "The problems posed constitute a gentle introduction to types of complex situations (e.g. scheduling, optimisation of flow, searching for computer files, studies of the migration of populations, etc.) that many pupils will encounter in later study, particularly in the areas of management and computer sciences. This topic sensitises students to algorithmics, and, by showing that graph theory is a powerful tool for modelling, allows for a new mathematical perspective on various situations." Basic elements of graph theory are taught, focusing on the resolution of problems.

The motivation behind these changes is to differentiate the orientations, to increase their value through differentiation and to develop a mathematical culture relevant to pupils' adult

lives and careers, that creates responsible citizens in a society where quantified information is omnipresent.

4.1.3 Taking into account the technological development

The French educational system was early to realise the importance of taking technological development into account. This awareness led to the introduction of a certain number of institutional initiatives, over a period of twenty years (too many for us to detail here). We will restrict ourselves to saying that since the counter-reform (that is to say since the early 1980s), the use of calculators has been explicitly mentioned in the curriculum, and that through successive versions of the curriculum, the incentive to integrate calculators and mathematical software has become greater, and that now, any type of calculator, including symbolic calculators can be used in mathematics at the *baccalauréat* level.

4.1.4 An increased focus on links with other scientific disciplines

The new curriculum also shows the increased importance given to the links between mathematics and other disciplines, especially scientific disciplines. Although this is evident in the syllabuses, the main change has been the introduction of a specific didactic organisation: the supervised personal work (TPE) in *première* and *terminale*.

It is in the syllabus of *terminale* S that this focus on linked disciplines is most visible. The exponential function was traditionally presented as the inverse function of the logarithm, but it is now introduced at the beginning of the academic year, for improving the coherence between mathematics and sciences teaching. The introduction of the exponential function starts with the differential equation f' =kf, whose study "can be justified by one or two examples, for instance radioactivity treated in physics, or by the search for derivable functions f, such as f(x+y)=f(x)f(y)".

TPE (*Supervised personal projects)* were introduced in 2000-2001 in *première* and were extended in 2001-2002 to *terminale*. Over one semester, small groups of students work on a collective project, using various resources, on a subject chosen by them that relates to national topics[5],. TPE must bring into play at least two disciplines including one which is essential to the students' orientation. The realisation of the project is supervised by teachers of the relevant disciplines with two hours per week reserved for the TPE in the students' timetable. The evaluation takes into account the production of the students as well as their written and oral presentations.

The curricular changes we have mentioned show how committed the institution is to dealing with the challenges that high school mathematics faces today. How were these changes received? Which strategies were used to help teachers understand their significance and implement them? How are they applied in classrooms today? It is too early to answer all these questions, but an evaluation would certainly indicate varied success. TPE, for instance, do not always work well: teachers claim that they are useful but also complain that they do not know how to handle them.

[5] The topics are differentiated according to the different orientations. The topics were in 2000-2001 and 2001-2002: Border, Arts, literature and policy, Representing the war, Memory/memories, the Barbarians, The city (L); The city, Leisure as a cultural practice, Companies and their territorial strategies, Reality and impact of quantified indicators, Elites, The newspaper industry (ES); Growth, Water, Images, Natural and technological risks, Sciences and food, Time, rhythms and periods (S). In "Terminale", two topics were common to the three orientations: Europe, Order and Disorder, and four were variables: The city, Border, Arts, literature and policy, Heritage and invention (L); Transformations of work, Elites, Companies and territorial their strategies, the city (ES); Growth, Space and movement, Image, Sciences and food (S).

4.2 The *lycée professionnel*

The main objective of the vocational branch is to allow for the professional integration of all children and to provide them with a minimum qualification as a worker.

There are three branches of study, which prepare students for a CAP, a BEP or a *baccalauréat*. In all branches of study, vocational training is based on general education, technological education and periods of practical apprenticeship in the professional world.

The branches leading to a CAP or a BEP prepare students for the same level of qualification; that of skilled workers, but they are different both in purpose and content. The CAP provides practical skill in particular areas, allowing for immediate professional integration. The BEP, on the other hand, teaches skills in the more demanding technological areas, for which professional integration requires higher qualifications. Many BEP students will continue their studies in order to take a vocational or technological *baccalauréat*. With a *baccalauréat professionnel* some students move on to tertiary education, essentially for a BTS (cf. below).

Mathematics is taught at every stages of the *lycée professionnel*, although the curricula are quite varied in terms of content and time allocated. All teachers in the *lycée professionnel* specialise in two subjects, therefore mathematics is taught by mathematics and science teachers.

In CAP, the teaching of mathematics represents 1.5 to 2 hours per week, per student, over two years. The main issue is to ensure students are taught techniques that relate to real professional problems. The varied professions are grouped in seven sectors of activities. The mathematical syllabus is divided into units, parts of them common to all sectors.

In BEP, the teaching of mathematics represents 2 to 3 hours per week, per student, over two years. There are two main areas of activities: industry and Tertiary. Students have to consolidate the mathematics they learnt in *collège* and acquire a few new notions:

- arithmetical and geometrical sequences
- common functions
- descriptive statistics
- basics of financial mathematics (tertiary sector only)
- two-dimensional vector geometry, space geometry and trigonometry (industry sector only)

In *baccalauréat professionnel*, the teaching of mathematics represents 2 hours per week, per student over two years. Again there are two main areas of activities: industry and tertiary. Students have to consolidate the notions encountered in BEP and acquire some new ones:

- equation and inequation of degree 2
- derivation and integration
- logarithm and exponential
- two variables statistical series
- differential equations
- discrete probability and normal distribution (industry sector only)
- mathematics for management (tertiary sector only).

A specific pedagogical device named PPCP (*Projet Pluridisciplinaire à Caractère Professionnel*), a multidisciplinary project with professional orientation, was set up a few years ago, for use in BEP and *baccalauréat professionnel* programmes. PPCP is a project that involves teachers from different disciplines, both professional and academic, and sometimes several classes of the same school. The goal is for students to work on a real professional question and to co-ordinate various aspects of their teaching. This is the vocational equivalent of TPE in *lycée d'enseignement général and technologique*.

5. ORGANIZATION OF THE TERTIARY LEVEL

The basic principles for higher education are fixed by the law of January 1984. The Ministry of Higher Education is responsible for all education in public establishments Nevertheless, since 1968, higher education establishments have autonomy in teaching, academic content, administration and finance. The so-called "Savary" law in 1984 re-defined this autonomy and it is now exercised in accordance with national regulations governing higher education and four-year contracts held with the State. This contractual policy aims, on one hand, to give true autonomy to universities and, on the other, to allow the State to fully exercise its responsibility for the co-ordination of activity in higher education. Each establishment draws up a development plan responding both to national objectives and to local training and research needs. This plan is addressed to the appropriate department of the Ministry, and then negotiated.

As the representative of the Minister for Higher Education, the *recteur* plays an administrative role in the direction, management and stimulation of institutions of higher education. He supervises the autonomous institutions.

A student with any type of *baccalauréat* can theoretically enter any type of university within his geographical district. Therefore the *baccalauréat* is not only the final examination of secondary school but also the first examination of tertiary education.

In order to understand the policy issues at stake in university level teaching in France, it is necessary to have an overview of the overall organisation of the tertiary education sector. Universities in France do not represent the only education options in post-secondary education.

Beside the universities, the tertiary education system offers various parallel institutions. There are three different main types[6]:

- Preparatory School to *Grandes Écoles* (*Classes Préparatoires* abbreviated to CPGE in the rest of the text);
- Advanced Technician Training (*Brevet de Technicien Supérieur* - BTS);
- University Institute of Technology (*Institut Universitaire de Technologie* - IUT)[7].

Each structure is organised differently, especially regarding policy issues.

5.1 Les classes préparatoires

The CPGE (*Classes Préparatoires aux Grandes Ecoles*) represents a cycle of two years of study. There are two main types of CPGE[8]:

- 'scientific': preparing students for the competitive admission examinations to engineering schools (*Grandes Écoles*[9]); and
- 'business': preparing students for the competitive admission examinations to business schools.

[6] We will not describe here other systems that are numerically unimportant.

[7] IUT are part of university structure, but with a specific remit, which will be detailed in a separate section.

[8] In fact it is a bit more complicated, there is quite a variety of 'scientific' CPGE, more specialised in mathematics, physics, computer, engineering sciences or biology/geology, etc. There are also less numerically important CPGE, in literature, or veterinary. For simplicity, we will not get into details.

[9] These range from Polytechnique, École Normale Supérieure, École Centrale, École des Mînes, etc., with an international reputation and a historical background dating from the French revolution, to more obscure highly specialised engineering schools.

The CPGE are located within the *lycées* (secondary schools) in every big city in the country. Classes have between 30 and 45 students. This system trains those students who are destined to become the élite of the country; it is therefore highly selective. At each *lycée*, a board of teachers and administrative managers selects the students who have applied, using evaluations made during the last two years of secondary school. There are no geographical conditions for admission and consequently there is competition between the different *lycées*. Globally the teachers teaching in CPGE are the 'best' of secondary school teachers. Most of them have been trained in one of the *Écoles Normales Supérieures*. These teachers have no obligation to undertake research, and most of the time, are not connected with research. As in secondary schools, the programmes are national, are determined by the government, and reflect the content of the competitive admission examinations of the *Grandes Écoles*. Each teacher decides on the type of assessment to be used during the year, but it is usually very intensive. Admission to the second year of preparation can be refused if the board of teachers judges that the results are too weak, but that is quite rare. Some students may also give up after a few weeks. At the end of the second year, if a student does not pass a competitive admission examination, or does not pass the one she or he wanted, she or he can be authorised to repeat (but this can be done only once). Students failing in this system usually continue their studies at a university.

Most engineers in France have been trained in this system. One advantage of this is that it ensures an initial high-level training in mathematics and physics/chemistry by highly qualified teachers. On the other hand, the training is purely academic and oriented toward selection. It therefore provides very little professional motivation or opening/experience. However this weakness has addressed by recent minor changes.

After CPGE, a variety of *Grandes Écoles* in engineering and business are available. Some are run by the state; some are (semi-) private. There is a wide range of levels of qualification on offer and the content of the teaching is very varied. Some schools are quite specialised and oriented toward a specific type of job, others offer a very general curriculum and, in some cases, the possibility of research activities in private or state organisations including universities. Most of the schools have an alumni association[10] that, as the essential organisation providing jobs to students, can be very influential in the design of the curricula. Most of these schools have a curriculum organised over three years. Each school decides on the number of students it can take each year. However, creating or abolishing a *classe préparatoire* is a decision taken by the government, one that can be influenced by local politicians in collaboration with the executive staff of a *lycée*.

Alongside this system there are a few engineering schools which enrol students for 4 or 5 years straight after the *baccalauréat*.

5.2 Brevet de Technicien Supérieur

This is a specific two-year training course for advanced technicians with various options in tertiary (selling, trilingual secretarial, etc.) or secondary activity (building, electricity, etc.). Like the CPGE, this structure is part of the *lycées* structure of most big cities (some specialisations are only offered in a very few places). Classes typically contain from 30 to 40 students. Admission is subject to a decision taken by a board of local teachers and administrative leaders, based on evaluations of the candidates made during the last two years of secondary school. The teachers are secondary school teachers, who spend only part of their teaching hours in these classes (unlike the teachers in CPGE who are fully employed there). The programmes are totally determined by the state. Creating or abolishing a class is a

[10] This network of ex-students is essential to ensure jobs for graduating students.

decision taken by the government. Assessment during the year is the responsibility of each teacher. Admission into the second year is decided by the board of teachers. At the end of the two years' training, students sit a national not competitive examination. The diploma they receive entitles them to find a job as an advanced technician in their speciality. Nevertheless, some students continue studying, either at university (especially in the IUP, cf. below) or, for a few of the best ones, in a *Grande École*. This system is very close in its organisation to the secondary school system but it has more connections with industry and has various professional partners. Although it is less prestigious than the *Grandes Écoles*, it offers students good employment opportunities and it also represents a smooth structural change with secondary school (small group classes, same institution) especially when compared to university (large lecture groups, tutorials, necessity to change city, etc.). Many students, especially from a lower social background or vocational and technological secondary education, prefer this system to the university system, but admission in the first year is subject to quotas and is therefore difficult to gain.

5.3 Institut Universitaire Technologique

This two-year training lies between secondary school and university. It is attached to a university, but in many ways it is independent of it. Teachers are recruited locally and are either secondary school teachers or university-type teachers (half teaching, half research work). Classes contain between 30 and 40 students. Admission is subject to a decision taken by a board of local teachers and administrative leaders essentially based on assessments made over the last two years of secondary school. The programmes are designed by each IUT according to nationally imposed schemes and are subject to four-yearly approval by the government. The schemes imposed by the State provide the main guidelines in terms of programmes, curricula, content in each subject, number of hours, assessment, relation with industry, etc. Therefore local innovation is real but limited. The decision to create a new IUT is local but subject to national approval. Moreover, every four years, if the IUT wants to remain in existence, the contract with the government has to be renewed and if the national guidelines have changed the preceding four years, that will influence the new proposal.

Student assessment over the two years of the course is organised according to the original project guidelines approved by the government. At the end of the two years, on the basis of the assessment criteria, the local board of teachers decides who will be given Diplomas. These Diplomas are national and entitle the student to find a job as an advanced technician in a specific field corresponding to her or his specialisation. Some students continue to study, either in a university (especially in IUP, cf. below) or, for very few of the best ones, in a *Grande École*.

5.4 Universities

Until 2003, the university system was organised in three cycles. The first cycle was two years long. At the end of it, the students who passed the examinations were given a diploma called the DEUG (*Diplôme d'Etudes Universitaires Générales*). The second cycle was also two years long. The first year examination was the *Licence*, the second was the *Maîtrise*. The third cycle started with one year of initiation to research that lead to a diploma called the DEA (*Diplôme d'Etudes Avancées*). This was followed by the doctorate, which was theoretically two to three years long. Since 2003, a reform, aiming at a European standard, has gradually changed the system, into what is known as the LMD system. In this new system there are still three cycles. The first cycle is three year long and finishes with the Licence. The second cycle is two years long. This constitutes the main innovation as it replaces the *Maîtrise* and the DEA, with the Master1 and Master2. The third cycle now comprises only the doctorate.

Moreover, new structures called IUP (*Instituts Universitaires Professionnalisés*) have recently been created within universities. They offer the same type of diploma with more professionally-orientated curricula and closer relations with industry. These structures are increasingly popular among students, especially of a lower social background, or who have had a vocational and technological secondary education (there are about 350 such structures in France now).

A student who has passed any type of *baccalauréat*, can gain admission to the first year of a university within a geographical district. In theory, there are no quotas. Some students enter a university outside of their geographical district, but this is exceptional and subject to a local decision. Admission to higher levels is subject to decisions taken by the appropriate board of local teachers according to the results obtained in the preceding years or cycles. From the first year, a student has to choose his specialisation (sciences, literature, social sciences, economics, etc.) with its limited range of options.[11] The specialisation becomes narrower as they progress through the three cycles.

There are national guidelines for each type of specialisation[12] with regards to programmes, curricula, content, hours in each discipline and the balance between small group tutorial and big group lectures, assessment, etc. Nevertheless, each university is responsible for the precise design of its programme. The different proposals, initially discussed and designed within departments, have to be approved at different levels within the university. Then, the university centralises the different propositions in a programme that has to fit the national guidelines. This programme is presented, in order to be approved at a national level, in a contract that each university signs with the National Ministry of Education and Research every four years. The general designs of the curricula are quite similar from one university to another, although there are often significant differences in the second and third cycles. Each department is responsible for the design of the part of the curriculum corresponding to its discipline even if the decisions are usually taken in collaboration with other departments. Finally, each teacher or group of teachers is responsible for the specific choices of content, material and organisation within the framework of the contract signed by the university and the state. The diplomas granted by the universities are national.

Generally, students choose their university for geographical reasons, therefore the differences in quality between universities are not globally significant. However, quality may vary in some specialisations, and Parisian universities traditionally attract the best teachers and students, whereas small provincial universities offer limited choices. Originally each *académie* had one to three universities (depending on the separation between sciences, social sciences and literature) of various sizes, located in the main city of the *académie*. In the last 20 years, a national political decision has led to an increasing number of students earning a *Bac*, and therefore entering university. Because of this, there has been local political pressure to create universities in the smaller towns of the *académies* (the buildings are partly financed by the local political authorities). Starting as entities dependent on the main university, offering teaching only at the first cycle level, these universities rapidly became more or less independent universities, offering an increasing variety of teaching at different levels and developing research departments. This was made possible by a general political decision taken by the French government in the early 1980s regarding the 'decentralisation' of the administrative structure at various levels, including those regarding education. In particular, finances for the building of universities are now dependent on the *Régions*, not on the State.

Teachers in universities are mainly part-time researchers holding tenured positions as civil servants. There are two categories: *Maîtres de Conférences* (lecturer) and *Professeurs*

[11] This is different from a system in which a student can choose any combination of two or three different major or minor subjects. In the French system the combinations are already decided in fixed proportions.
[12] In particular the different types of specialization that can be offered are imposed by the government.

(professors). Both categories have the same obligations in terms of teaching (192 hours per year). Each year, every presents the government with an ordered list of new teaching jobs it would like to have available for the following year, based on the wishes of each teaching and research department. On the basis of these proposals, the government decides to create a certain number of jobs in each university. Once the government creates a job, the university is in charge of the employment of the teacher subject to specific rules. The potential candidates have to be approved by a national commission made of teachers in the same field (2/3 are elected, 1/3 are designated by the government). Anybody who has been approved at the national level can apply for any job at the level of his qualification (lecturer or professor). Appointment decisions are then taken locally. Each university has specific recruiting commissions for each main subject. The decision to appoint is mostly based on research criteria.

Some teachers are full time teachers detached from the secondary school structure holding tenured positions as civil servants in a university. Their number increased quite rapidly in the 1980s, with the increasing number of students in the first cycles. This was the result of a national orientation taken against the wishes of most teacher-researchers. The strategy was economical for the government -one full-time secondary teacher required teach twice the number of hours of a teacher-researcher, and paid a similar salary[13], replaced two of the latter.

Some teachers are employed on the basis of short-time contracts. They are usually students preparing for their doctorate or just completing it.

Some teachers are employed on an hourly basis. Although globally this represents only a low percentage of teaching hours, it covers a variety of teaching situations, in particular significant proportion of the hours spent teaching secondary subjects in a specialisation (e.g. English for scientists, or mathematics in social sciences, etc.).

Connections between universities and industry were traditionally somewhat rare in France. Universities represent the academic world and usually have very few contacts with industry. Nevertheless, in recent years, many students with a *baccalauréat* could not get into a BTS or an IUT and therefore entered a university as their last choice. A university offering a general education could not provide the rapid professional training they desired. This partly explains the fact that while the number of students in the first cycles of universities has increased significantly in the last 15 years, the number of students in the following cycles has remained more or less the same, or even decreased in some academic subjects. In response to this new situation, universities introduced a new type of training with closer connections with industry, especially the IUP. Moreover, there are now more ways of entering an engineering school from a university, without going through *the classes préparatoires*.

5.5 Some figures

In 2000/2001 the total amount of students in tertiary education in France was just over 2 millions (out of a global population of roughly 60 millions inhabitants).

The repartition is shown in Table 5:

University	67%
BTS	11%
IUT	6%
CPGE	4%
IUFM (see below)	4%

[6] It is not rare for a secondary school teacher employed in a university to have, or be preparing for, a doctorate.

Engineering schools	3%
Others	6%

Table 5 *– Repartition in tertiary education*

Table 6 shows the repartition in first year of tertiary education according to the type of *baccalauréat*.

	Scientific bac série S	*Baccalauréat General*	*Baccalauréat Technologique*
University	57,9%	62,4%	19,1%
BTS	7,3%	9,3%	44,9%
IUT	14,6%	11,2%	9,1%
CPGE	19,1%	12,6%	1%
Others	0,2%	4,5%	25,9%

Table 6 *– Repartition in first year of tertiary education*

5.6 Conclusion

This system may be perceived as rigid and extremely complex. For instance, the system of *Grandes Écoles* is a heritage from the French Revolution and centralisation was introduced by Napoleon. Nevertheless, this system, although archaic in some sense, is what has so far preserved the teaching, by specialists, of academic subjects like mathematics and physics in engineering and business schools. It is not rare for a student, after graduating from an engineering school, to start a doctorate in fundamental research in physics, computer science or applied mathematics.

It is clear that the *Classes Préparatoires* offer the best quality of mathematics teaching, and it is there that the students the most able and most motivated. Mathematics teachers in these institutions have a very high level of qualification (even if they are not usually in contact with mathematics research). They also have the best salaries in the French educational system and are very respected. Through their official representatives in the UPS, they can influence changes in curricula and enjoy a great liberty in their pedagogical choices as long their students do well in the national competitive examinations. The IUT and BTS are small structures in which teachers have direct impact on policy matters at the local level, even if they are tied to national guidelines (that can be more or less restrictive). The selection of students on entrance ensures a good rate of success. Moreover, in these two institutions, mathematics is usually a secondary subject as the training is orientated directly towards a profession. Therefore the connection between mathematics and other subjects and industry are an important issue that each mathematics teacher must address. It is certainly within universities, that mathematics teachers encounter the most difficulty. Unless they work in one of the few, highly regarded universities where teaching and learning is at the highest level, they may have to face large audiences of students lacking professional or academic motivation. In the first two years especially, the rates of success are very low and the level of students very heterogeneous. Overall the teaching conditions are poor. Furthermore, the size of the structure does not favour individual initiative and most teachers' influence on policy issues is very limited. Moreover, a university teacher is generally also a researcher and is only evaluated on his research work. Consequently, many university teachers spend more time and energy on their research (which represents their real career) than on their teaching, for which they get little satisfaction either in terms of students' motivation, or in terms of professional recognition from their institution.

6. TEACHERS' TRAINING[14]

Pre-service teachers' training was substantially modified in the early 1990s for several reasons: new needs in education, and in recruitment, evolution in educational policies, the necessity to promote the profession, and the development of research in education, especially regarding mathematics. The "rapport Bancel" (named after the *recteur* Bancel director of the commission that wrote the report for the Ministry of Education), published in 1989, launched the bases for the reform. In the same year, the new law on education introduced new requirements regarding the development and improvement of teacher education. In 1990, three pilot University Institutes for Teacher Education (IUFM: *Institut Universitaire de Formation des Maîtres*) were created. And in 1991, one IUFM by *académie* was created. The reform aimed at more coherence between elementary and secondary education, a five-year teacher education process for all types of teachers, more coherence between academic and professional training as well as between theory and practice, more links between research in education and teachers' training.

There are three main types of teachers: primary school teachers (PE *professeurs des écoles*), general and technical secondary school teachers (PLC *professeurs des lycées et collèges*) and teachers for vocational education (PLP *professeurs des lycées professionnels*).

They are all recruited at the same level (except for *agrégés*, see below), namely the *Licence* (three years after the *baccalauréat*), through a selective national competitive examination, and therefore have the same salaries. They usually take a year in the IUFM to prepare the examination (but this is not obligatory). The number of positions offered each year for each different examination is fixed by the state taking into account the national needs and the demography of the teacher population. The ratio of candidates for positions varies with regards to the type of teachers, the discipline and the general economical context, but it is usually quite selective (up to 10 candidates for 1 position).

The IUFM are university institutes although they cannot deliver diplomas. Trainers are either university teachers (about a third) or primary and secondary teachers, either part-time or full time in the IUFM. The IUFM propose preparation for students who have a licence (that they passed in a university). Although for secondary school teachers the preparation is mostly or even entirely supervised by universities. Moreover, students can follow a preparation in a private institute (mostly for primary school teachers), or simply take the examination without any institutional preparation. One they have passed of type of examination, they have to follow a one-year part-time training in the IUFM where they have be assigned. Once they validate their one-year training (which happens in most cases) they get a tenure position as civil servant and are assigned by the state or the *académie* to a school, if possible in accordance with their geographical wishes.

6.1 Primary school teachers

Primary school teachers teach all disciplines. The examination candidates may have any type of *licence*. However, in recent years, specific curricula in educational science, and optional modules orientated towards education have been developed at most universities. The level in mathematics is varied, but generally quite low.

[14] For more information see:

Comiti C., Ball N.: 1996, 'Preparing teachers to teach mathematics: A comparative perspective'. In A. Bishop & al. (eds), *Handbook of Research in Mathematics Education*, *1123-1154*, Dordrecht: Kluwer Academic Publishers.

Henry, M., Cornu, B.: 2001, 'Mathematics teachers' education in France: from academic training to professionalization', In D. Holton & al. (eds), *The Teaching and learning of Mathematics at University Level: an ICMI study, pp. 481-499,* Dordrecht: Kluwer Academic Publishers.

Although the examination is national, it is different in each *académie*. As a matter of fact, a primary school teacher is attached to a specific *département* in France. The structure of the exam is always the same: the first part is written, and consists of a 3-hour test in mathematics and a 4-hour test in French (although this is subject to current reform). The mathematics test is divided into two parts. One consists of general mathematical knowledge, while the other concerns didactics. Students have to analyse pupils' productions and pedagogical material. During the year of preparation at the IUFM they are trained both in mathematics (mainly those who had followed non-scientific studies) and in the didactics of mathematics. They are also given short practical training sessions in schools.

In recent years, about 10 000 primary school teacher positions were offered every year, for about 60 000 candidates. There is a specific examination for primary school teachers in private education. In 2000/2001, about 315 000 primary school teachers were in public education, as opposed to 44 000 in private education.

Once they have passed the examination, students teachers take various types of courses in education, psychology and didactics and undertake practical training in schools. They must also write a professional dissertation, a key element in their training with regards to the articulation between theory and practice. They spend part of their time in the classroom alone, responsible for teaching pupils. They are supervised by an experienced teacher in the same school, and receive formative and assessment inspections from the IUFM's staff. Some of the theoretical training is related to the didactics of mathematics, but this usually represents a small part of the total training time.

In recent years, quite a few students who have failed the competitive examination have, nevertheless, been asked, usually during the first weeks of the academic year, to take up a position as full time teacher (those with the highest marks are called first). In this case, they are entitled to receive their training year after this year of experience, if they have not given up at this stage!!! However, all recently recruited primary school teachers have received a one-year training where the articulation between theory and practice was an essential element.

6.2 General and technological secondary school teachers

This type of teacher is trained to teach one specific discipline at all seven levels of lower and higher secondary school, in all different specialised series of general and technological education. In the case of mathematics, this represents quite a wide range of syllabuses. There are two types of competitive examination: CAPES (*Certificat d'Aptitude au Professorat de l'Enseignement Secondaire*) and *Agrégation,* only accessible after the *maîtrise* (four year in a university). These lead to two categories of teachers: *certifiés* and *agrégés*, who are entitled to teach in the same type of classes, *certifiés* have to teach 18 hours a week, while *agrégés* only have to teach 15 hours and are better paid. After a few years, *certifiés* are entitled to take a specific examination in order to become *agrégés*, called *agrégation interne*. In certain cases, students with a licence can teach in secondary education without any specific teaching qualification, on a temporary basis. After a few years, they are entitled to take a specific examination in order to become *certifiés*, called CAPES *interne*. In recent years, the number of non-qualified temporary positions has been reduced (about 15 000 against the 369 000 tenure positions in 2000/2001).

For mathematics, in recent years, about 1 000 positions were offered at CAPES for about 8 000 candidates, and about 300 positions offered at *Agrégation* for about 2 000 candidates.

There are also specific examinations for private education. In 2000/2001, there were about 310 000 secondary school teachers in public education, and 77 000 in private education.

Usually students follow one year of preparation in order to take CAPES or *agrégation*. For *agrégation*, such preparation is offered either in universities or in the *Ecoles Normales*

Supérieures (see below). The preparation for CAPES is, in theory, supervised by IUFM but, in practice, is mostly organised by universities. Both examinations start with a written part (three 6-hour tests for *agrégation*, two 5-hour tests for CAPES in mathematics) that is strictly academic. Those who pass the written part are entitled to take the oral part, which consists of two one-hour presentations (for each oral presentation, the candidate has to choose between two subjects imposed by the examiners, from a list of about 80 themes known in advance). While for *agrégation*, the oral presentations are quite academic and include content beyond the level of secondary education, one of the two CAPES presentations requires the candidate to demonstrate three exercises relating to a theme selected by the examiners, and explain the reasons why he choose them. The preparation of this oral presentation includes selecting exercises from a variety of textbooks, analysing their potential for the teaching of the theme, their relevance with respect to curriculum, finding different ways of solving a problem and also different ways of formulating and organising the questions in an exercise, etc.

During their university studies, students are trained in mathematics in a way that is still influenced by Bourbaki. They are introduced to algebraic structures, formal definitions for basic concepts of calculus (such as limit), formal proofs of fundamental theorems and, theories such as Riemann integral. The curriculum focuses on algebra and analysis. In most universities, geometry, which plays an important role at secondary level, nearly disappears, with the exception of some analytical geometry linked to linear algebra. In tthe third year (licence) there is a qualitative jump in the level of abstraction with courses in: general topology, differential calculus in normed vector spaces, integration, functional analysis, set theory, abstract algebraic structures and (but often not compulsory) probability, affine, Euclidean and projective geometry. Courses in computer technology are offered from the first year in every university, and students are often introduced to CAS software such as Maple, Mathematica or Matlab. However, connections with other disciplines, applications and modelling, discrete mathematics, statistics, numerical analysis are usually absent from the curricula up to the *licence*, and only optional at the *maîtrise* level. Thus, most students in mathematics preparing CAPES or even *agrégation* have a pretty abstract view of mathematics, even with the changes that have been introduced in recent years. The preparation of CAPES or *agrégation* offers students an opportunity to develop more synthetic and integrated views about mathematics along with some reflexive thinking about mathematical objects. Yet, it is far from being a professional training in education, even if some short observation sessions in secondary school are offered (but these are not compulsory).

Once they have passed the examination (either CAPES or *agrégation*), students become teachers-students and have to follow a one-year training course, that they need to validate in order to get a tenure position as civil servant (very few do not pass the validation). This training is based on an articulation between theory and practice. Teacher-students spend about a third of their time (6 hours per week) teaching in a *lycée* or a *collège*, and are entirely responsible for their teaching all through the year, under the supervision of a senior teacher from the same institution. They are also inspected the IUFM's staff, both from a formative and assessment point of view. Their theoretical training is based on general education, psychology and pedagogy on one hand and didactics, epistemology and the study of secondary school curricula in their discipline, on the other. They must also write a professional dissertation, a key element with regards to the articulation between theory and practice.

6.3 Teachers in lycée professionnel

As has been said before, teachers in *lycée professionnel* (PLP) have to teach two subjects, therefore mathematics teachers do not exist, instead there are math and science teachers. Like other secondary school teachers, PLP are recruited through a national competitive examination CAPLP, after a licence in mathematics, physics or chemistry. Competition is quite fierce, as, in recent years only about 300 positions have been offered for nearly 3 000 candidates. The examination consists of a written part with two 4-hour tests, including one in mathematics, and, after a first selection, an oral part with two presentations on imposed themes taken at random among a list, one in mathematics, one in science.

A one-year preparation for CAPLP is offered in some IUFM. Some students follow the preparation for CAPES either in mathematics or in science and take both CAPES and CAPLP.

The one-year training course is quite similar to that offered to other types of secondary school teachers, with two main differences:

- Students-teachers in PLP are usually more trained in one of their two subjects and therefore need some complementary teaching in the other discipline.
- They need to become familiar with the practices common in a vocational institution (there will differ from those they have experienced) and acquire practical information about a world totally stranger to their own practice as students.

6.4 In-service training (secondary school teachers)

In-service training is organised through the *rectorat*, mostly via the *inspecteurs*, on the basis of the annual plan established by each *académie*. A call for contributions is sent to every school and to the IUFM and universities. There are also some orders from the *Recteur*, concerning for example, new curricula or pedagogical devices. All possible training courses are published in a catalogue and on a web site, and teachers have to apply for them. If they are accepted, their training time is deducted from their teaching time and their timetable has to be adapted in consultation with the head of their school. The duration and frequency of the training sessions can be varied. It is usually a set of 3-hour sessions held on the same weekday over a few weeks during the year, or it can be an intense session taking place over a week or a few days. Preparation for CAPES or *agrégation internes* is considered as in-service training and can be subject to a reduction of teaching time.

Any teacher can propose a training session on practically any subject, although he must follow a certain framework. In mathematics, the IREMs (Instituts Universitaires de Recherche en Mathematiques) are playing an important role concerning in-service teachers' training.

In-service training is a right, but it is not an obligation. Therefore, teachers have very different levels of engagement with it. It would be difficult to give an evaluation in this context.

6.5 Ecoles Normales Supérieures

The *Ecoles Normales Supérieures* are a prestigious old French heritage dating from the 1789 Revolution. Of the four main such institutions, the oldest is in Paris (known as *La Rue d'Ulm*), another is in Cachan (a Parisian suburb) with an annex in Britany near Brest, and two are in Lyon (one for science, one for literature and human sciences).

They are supposed to train the elite of French teachers and researchers. In mathematics, there are less than 100 positions every year. Most students gain access is through taking very selective competitive examinations after two years in a *Classe Préparatoire*, although it is possible to enter via university. Students are civil servants, and as such are paid during their

studies: in exchange they sign an engagement to work for the State over a period of ten years (including their 4-year training). Students mainly follow university studies, but also have high level complementary course and a specially designed training course for *agrégation*. Most students follow on with a doctorate and therefore become university teachers or full-time researchers, but they may also become teachers in *Classe Préparatoire*.

7. CONCLUSION

In Education, as in many areas, the French system is very centralised and standardised by government control. Even if the educational system in France may appear to have gained more autonomy in recent years, individual initiatives and originality have to fit national guidelines and are subject not only to local but also national control. This tends to unify everything.

Yet, although it is centralised, the system is under democratic control. Indeed, the decision-making process at the top of the system is controlled by different counter-power organisations: the unions of course, but also professional and research associations as well as academic societies, which may influence decisions more specific to the content and curriculum.

There are six quite powerful associations involved in mathematics, federated within the CFEM (French Commission for Mathematical Education), which is the French representative of the ICMI (International Commission for Mathematical Instruction). These are:

- the Association of Mathematics Teachers (APMEP)
- the Institutes of Research in Mathematical Education (IREM)
- the Association for Research in Mathematical Education (ARDM)
- the Association of Teachers in Classes Préparatoires (UPS);
- the French Mathematical Society (SMF);
- the French Applied Mathematics Society (SMAI).

Members of these organisations act as political consultants, and their work is usually taken into account by politicians responsible for making decisions in this field.

For instance, a core part of the French educational system, is the national curriculum. A national council for curriculum (CNP) designs the general curricular organisation and prepares specific guidelines for each discipline. In addition, for each discipline, there exists a group of experts in this particular field who work out the syllabuses. These syllabuses are then submitted for approval to the CNP and to various other authorities.

In 1999, following a request made by various associations of mathematicians and mathematics teachers, Claude Allègre, then the French Minister of Education, created in 1999 a ‘commission of reflection on the teaching of mathematics’ (CREM). This commission, first presided over by Jean Pierre Kahane, and now by Jean-Christophe Yoccoz, has been asked "to propose perspectives for the evolution of mathematics teaching, its better connection with the teaching of the other disciplines and for the correlative necessary transformation of teacher education”. It has 18 members and aims at a longer-term reflection on the teaching of mathematics than the CNP and the group of experts, but it must connect its work with that of these two groups, which are represented in it. Until now, this commission has produced 7 reports related respectively to mathematics and computer sciences, geometry, computation, probabilities and statistics, pre-service and in-service teacher education, vocational education,

the relationships between mathematics and other scientific disciplines. These can be accessed, with the appendices which accompany them, on various web-sites.[15]

[15] Among these, the web site of the Ministry : www.education.gouv.fr, and the web-sites of the different mathematics associations.